

In this lattice picture a ray corresponds to a ray ending up exactly on lattice point removed from its origin position.

- ① pick a direction vector
- ② calculate reflection off 3 sets of orthogonal plane
- ③ calculate travel distance between successive hits on a single set of planes
- ④ assign a loss/cm number

This looks amenable to an analytic treatment.

5-Sep-97

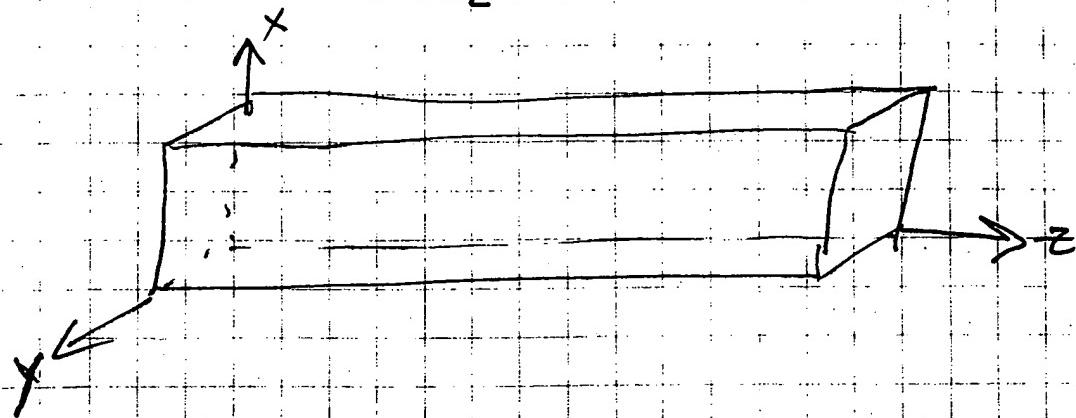
use direction cosines to parameterize ray direction

$$(\cos \theta_x, \cos \theta_y, \cos \theta_z) = \frac{(\text{RND}_1, \text{RND}_2, \text{RND}_3)}{\sqrt{\text{RND}_1^2 + \text{RND}_2^2 + \text{RND}_3^2}}$$

let  $\Delta x$ ,  $\Delta y$ , and  $\Delta z$  denote slab dimensions or plane spacing.

6-Sep-97

From the point of view - it doesn't matter what position a ray is launched from, only its direction, because launch position has no impact on spacing between plane strikes.



Treat cants on surface perturbatively.

2 questions:

(1) How big do cants have to be to eliminate all parasitics?

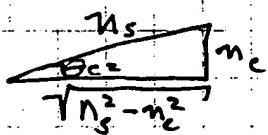
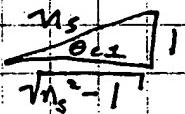
(2) For a rectangular slab, how close to slab index does cladding index have to be to eliminate all parasitics.  
This question will be easiest to answer for a zero loss parasitic

set z face incident angle equal to  $\theta_{critz} = \sin^{-1}\left(\frac{n_c}{n_s}\right)$  and the x face hit and y face hit also =  $\theta_{critx} = \sin^{-1}\left(\frac{n_c}{n_s}\right)$

make this argument more rigorous!  
Now work with direction cosines

$$\theta_{c1} = \sin^{-1}\left(\frac{1}{n_s}\right)$$

$$\theta_{c2} = \sin^{-1}\left(\frac{n_c}{n_s}\right)$$



$$\cos \theta_{c1} = \frac{\sqrt{n_s^2 - 1}}{n_s}$$

$$\cos \theta_{c2} = \frac{\sqrt{n_s^2 - n_c^2}}{n_s}$$

$$\cos^2 \theta_{c1} + 2 \cos^2 \theta_{c2} = 1$$

$$\frac{n_s^2 - 1}{n_s^2} + \frac{2(n_s^2 - n_c^2)}{n_s^2} = 1$$

$$n_s^2 - 1 + 2n_s^2 - 2n_c^2 = n_s^2$$

$$2(n_s^2 - n_c^2) = 1$$

$$n_s^2 - n_c^2 = \frac{1}{2}$$

when can this no longer be solved

$$n_c = \sqrt{n_s^2 - \frac{1}{2}}$$

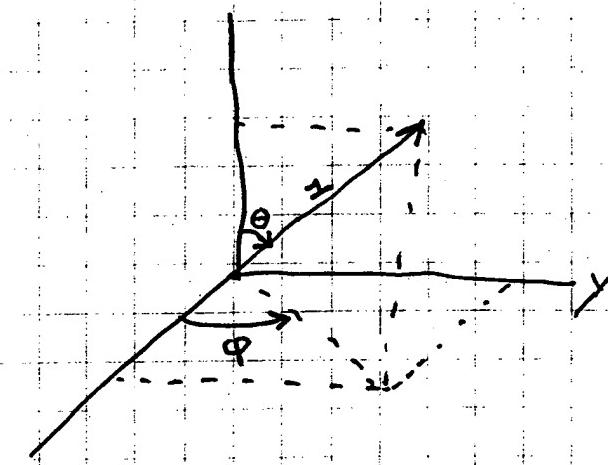
$$n_c = \sqrt{682^2 - \frac{1}{2}} = 1.677$$

for  $n_c > 1.677$  no zero loss  
parasitics exist

(This agrees with  
code: slab ASE 01. XCL  
prediction.)

Question 2 will be easiest to answer numerically by  
finding the angular width over which  
a parasitic exists for given gain and  
cladding indices.

7-Sep-97



$$x = \sin \theta \cos \phi = \cos \theta_x$$

$$y = \sin \theta \sin \phi = \cos \theta_y$$

$$z = \cos \theta = \cos \theta_z$$

$\theta_x < \theta_{x\text{crit}}$   
 $\theta_y < \theta_{y\text{crit}}$   
 $\theta_z < \theta_{z\text{crit}}$

to avoid O-loss  
parasitics

$$\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1$$

$$\cos \theta_x > \cos \theta_{x\text{crit}}$$

$$\cos \theta_y > \cos \theta_{y\text{crit}}$$

$$\cos \theta_z > \cos \theta_{z\text{crit}}$$

to avoid  
O-loss  
parasitics

$$\sin \theta_{x\text{crit}} = \frac{n_c}{\sqrt{n_s^2 - n_c^2}}$$

$$\sin \theta_{y\text{crit}} = \frac{n_c}{\sqrt{n_s^2 - n_c^2}}$$

$$\sin \theta_{z\text{crit}} = \frac{1}{n_s} \quad \frac{n_c}{\sqrt{n_s^2 - 1}}$$

$$\cos \theta_x > \frac{\sqrt{n_s^2 - n_c^2}}{n_s}$$

$$\cos \theta_y > \frac{\sqrt{n_s^2 - n_c^2}}{n_s}$$

$$\cos \theta_z > \frac{\sqrt{n_s^2 - 1}}{n_s}$$

to avoid  
O-loss  
parasitics

$1 = \cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z$  and this must be greater than

$$1 > \frac{n_s^2 - n_c^2}{n_s^2} + \frac{n_s^2 - n_c^2}{n_s^2} + \frac{n_s^2 - 1}{n_s^2}$$

to avoid  
O-loss  
parasitics

$$z > \frac{3n_s^2 - 2n_c^2 - 1}{n_s^2}$$

$$n_s^2 > 3n_s^2 - 2n_c^2 - 1$$

$$1 > 2(n_s^2 - n_c^2)$$

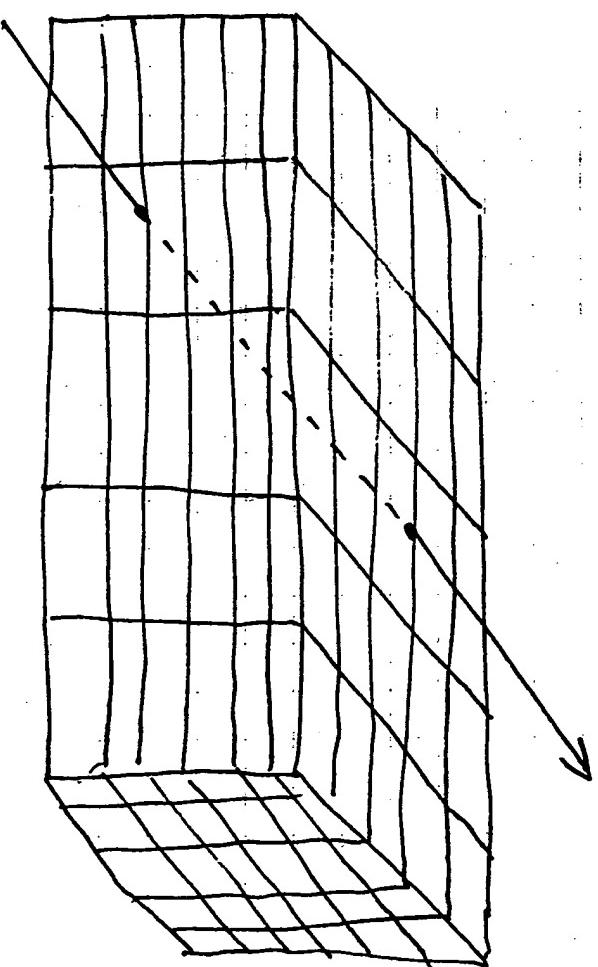
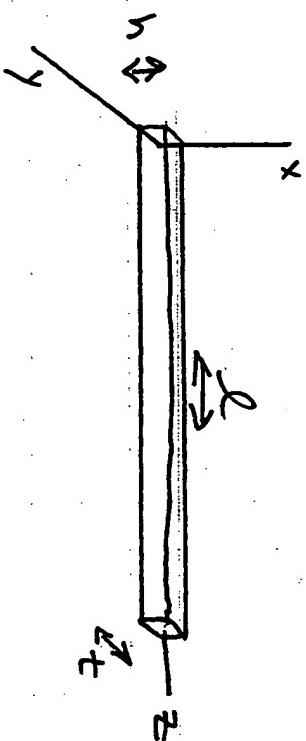
$$\frac{1}{2} > n_s^2 - n_c^2$$

$$n_c^2 > n_s^2 - \frac{1}{2}$$

$$n_c > \sqrt{n_s^2 - \frac{1}{2}} = \sqrt{1.82^2 - \frac{1}{2}} = 1.677$$

$n_s$  is slab index  
 $n_c$  is coating index

parametric progression direction can be generated  
using a method of images construction



fill space  
using slabs  
and  
slab images

- Define arbitrary ray direction using direction cosines  
 $(\cos \theta_x, \cos \theta_y, \cos \theta_z)$
- Gain of rays in given by  
 ~~$\frac{\text{Len(Refx)}}{\text{Len(Refy)}}$~~   $\frac{\text{Len(Refy)}}{\text{Len(Refz)}}$  &  $\frac{\text{Len(Refz)}}{\text{Len(Refx)}}$

$$g = \frac{(h/\cos_x)}{(t/\cos_y)} \quad \frac{(t/\cos_y)}{(L/\cos_z)}$$

where:

$\text{Ref}_i$  is the reflection coefficient for  $i$ -oriented planes  
 $L$  is a scalar specifying gain. Considering

zero-loss parameters correspond to those may directions that are confined by TIR at all three sets of planes

$$\left. \begin{array}{l} \cos \theta_x < \cos \theta_{x\text{-crit}} = \frac{\sqrt{n_s^2 - n_c^2}}{n_s} \\ \cos \theta_y < \cos \theta_{y\text{-crit}} = \frac{\sqrt{n_s^2 - n_c^2}}{n_s} \\ \cos \theta_z < \cos \theta_{z\text{-crit}} = \frac{\sqrt{n_s^2 - 1}}{n_s} \end{array} \right\}$$

where:  
 $n_s$  = slab index  
 $n_c$  = coating index

• since  $\Gamma = \cos \theta_x^2 + \cos \theta_y^2 + \cos \theta_z^2$   $\approx$  zero loss parameters exist

$$\Gamma < \frac{n_s^2 - n_c^2}{n_s^2} + \frac{n_s^2 - n_c^2}{n_s^2} + \frac{n_s^2 - 1}{n_s^2}$$

or

$$n_c < \sqrt{n_s^2 - \Gamma}$$

zero-loss parameters can be completely satisfied by choosing a cladding with refractive index large enough

$$n_c > \sqrt{n_s^2 - \Gamma}$$